LESSON 3.4a

Using the Quadratic Formula to Solve Quadratic Equations

Today you will:

- Solve quadratic equations $x^2 + bx + c = 0$ $(a = 1)$ using the *Quadratic Formula*

- Practice being a math translator

Core Vocabulary:

- Quadratic formula, p. 122

Solve $ax^2 + bx + c = 0$ for x

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$ Factor out and divide by a

$$x^2 + \frac{b}{a}x \quad = -\frac{c}{a}$$ Complete the square

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$ Factor left side, simplify right side

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$ Solve using square root

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Simplify

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

The Quadratic Formula

- What you get when you solve $ax^2 + bx + c = 0$ for x.
- Allows you to solve *ANY* quadratic.
- A generalized formula that gives you the *roots* of any quadratic.

Let $a, b, and\ c$ be real numbers such that $a \neq 0$.

The solutions for the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

COMMON ERROR

Remember to write the quadratic equation in standard form before applying the Quadratic Formula.

Solve $x^2 + 3x = 5$ using the Quadratic Formula.

SOLUTION

$$x^2 + 3x = 5$$ Write original equation.

$$x^2 + 3x - 5 = 0$$ Write in standard form.

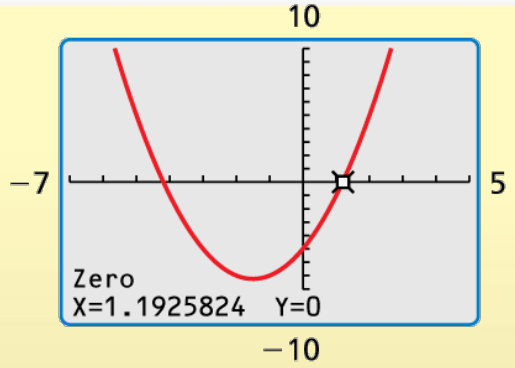
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)}$$ Substitute 1 for a, 3 for b, and -5 for c.

$$x = \frac{-3 \pm \sqrt{29}}{2}$$ Simplify.

So, the solutions are $x = \frac{-3 + \sqrt{29}}{2} \approx 1.19$ and $x = \frac{-3 - \sqrt{29}}{2} \approx -4.19$.

Check Graph $y = x^2 + 3x - 5$.
The x-intercepts are about -4.19 and about 1.19. ✓



ANOTHER WAY

You can also use factoring to solve $25x^2 - 20x + 4 = 0$ because the left side factors as $(5x - 2)^2$.

Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

SOLUTION

$25x^2 - 8x = 12x - 4$ — Write original equation.

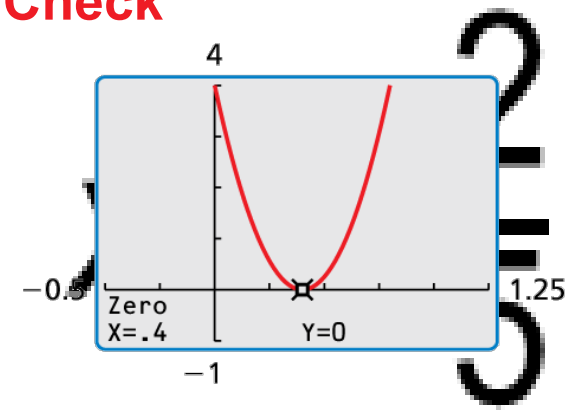
$25x^2 - 20x + 4 = 0$ — Write in standard form.

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$ — $a = 25$, $b = -20$, $c = 4$

$$x = \frac{20 \pm \sqrt{0}}{50}$$ — Simplify.

$$x = \frac{2}{5}$$ — Simplify.

Check



So, the solution is $x = \frac{2}{5}$.

Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

SOLUTION

$$-x^2 + 4x = 13$$ Write original equation.

$$-x^2 + 4x - 13 = 0$$ Write in standard form.

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-13)}}{2(-1)}$$ $a = -1$, $b = 4$, $c = -13$

$$x = \frac{-4 \pm \sqrt{-36}}{-2}$$ Simplify.

$$x = \frac{-4 \pm 6i}{-2}$$ Write in terms of i.

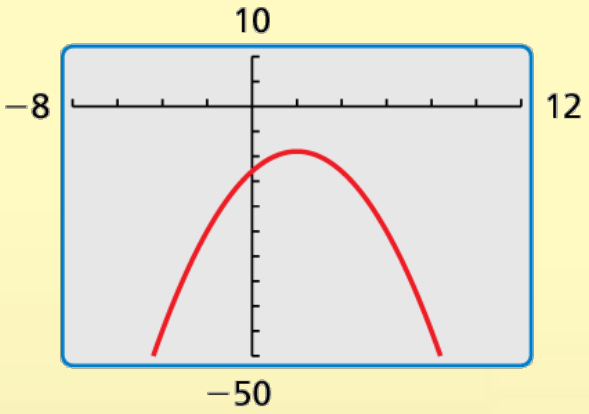
$$x = 2 \pm 3i$$ Simplify.

The solutions are $x = 2 + 3i$ and $x = 2 - 3i$.

COMMON ERROR

Remember to divide the real part *and* the imaginary part by -2 when simplifying.

Check Graph $y = -x^2 + 4x - 13$. There are no x-intercepts. So, the original equation has no real solutions. The algebraic check for one of the imaginary solutions is shown.



$$-(2 + 3i)^2 + 4(2 + 3i) \stackrel{?}{=} 13$$

$$5 - 12i + 8 + 12i \stackrel{?}{=} 13$$

$$13 = 13 \checkmark$$

Homework

Pg 127 #1-18, 59-60